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EAC manifolds with structure group G_2

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ABSTRACT

In this paper we will consider the deformation theory of compact G-manifolds, where G = G_2 We will prove that the moduli space of torsionfree G-structures is a smooth manifold.also proved smoothness of the moduli space on compact G-manifolds for any of the Ricci-at holonomy groups G_2 in a fairly uniform way.The arguments used here are geared to make it easier to generalise to the asymptotically cylindrical case in physics.

Keywords: *EAC manifolds, G_2manifolds, cylindrical.*

INTRODUCTION

A way to obtain irreducible compact G_2-manifolds is by gluing a pair of noncompact G_2- manifolds which are asymptotically cylindrical. A manifold is said to have cylindrical ends if it is homeomorphic to a cylinder outside a compact piece. An asymptotically cylindrical manifold is a Riemannian manifold with cylindrical ends for which the met-ric is asymptotic to a product metric on the cylindrical ends. Asymptotically cylindrical manifolds are easier to work with than arbitrary non-compact manifolds. many analysis results for elliptic operators on compact manifolds can be generalised to statements about asymptotically translation-invariant elliptic operators acting on suitable spaces of sections on an asymptotically cylindrical manifold. In some arguments it is helpful to impose a

Stronger condition, requiring the manifold to be exponentially asymptotically cylindrical (EAC).Given a pair of EAC G_2-manifolds whose cylinders match one can form a gener- alised connected sum by truncating the cylinders after some large but finite length and gluing them together. If the neck length is su_ciently large then the EAC G_2-structures can be glued to form a torsion-free G_2-structure on the connected sum. This is a gluing construction for compact G_2-manifolds. Kovalev proves an EAC version of the Calabi con- jecture to produce EAC Calabi-Yau 3-folds. By multiplying with circles reducible EAC G_2-manifolds are obtained, which can be glued to form irreducible compact G_2-manifolds different topological types from those constructed by Joyce.

Definition 1.1. Let X^6 be a compact manifold, and denote by t the R-coordinate on the cylinder X R . Let M be a Riemannian manifold with HOL(M) $\subseteq$ H and ρ α representation of H. The Lichnerowicz Laplacian on E_P is the formally self adjoint operator

$$\Delta_P = \nabla * \nabla - 2(D_p)^2(R): \Gamma(E_p) \to \Gamma(E_p)$$

where ∇ is the connection on E_p induced by the Levi-Civita connection on M.

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Definition 1.2. A G_2-structure' on X × R is cylindrical if it is translation-invariant and the associated metric is a product metric $g_\varphi = g_x + dt^2$

Definition 1.3. A manifold M is said to have cylindrical ends if it is a union of two pieces M_0 and M∞ with common boundary X, where M_0 is compact, and M_∞ is identified with X × R^+ by a diffeomorphism (identifying ∂ M_∞ with X × {0} X is called the cross-section of M.

Definition 1.4. A tensor field or differential operator on X ×R is called translation invariant if it is invariant under the obvious R-action on X ×R .

Definition 1.5. An asymptotically cylindrical manifold is a Riemannian manifold with cylindrical ends for which the metric is asymptotic to a product metric on the cylindrical ends

Definition 1.6. A metric g on a manifold M with cylindrical ends is said to be EAC if it is exponentially asymptotic to a product $g_x + dt^2$ metric on $X \times R^+$. An EAC manifold is a manifold with cylindrical ends equipped with an EAC metric.

Proposition 1.7. Let M_7 an EAC G_2 -manifold with cross-section X. Then

$$H_6^2\ (X) = A_6^2 \oplus E_6^2, H_6^4(X) = A_6^4 \oplus E_6^4$$

and the sums are orthogonal. Furthermore

(i) $H_6^2\ (X) \rightarrow H_6^4(X), [\alpha] \rightarrow *[\alpha]$ maps A_6^2 to E_6^4 and E_6^2 to A_6^4

(ii) $H^1\ (X) \rightarrow H_6^4(X), [\alpha] \rightarrow [\alpha] \cup [\Omega]$ maps A^1 to A_6^4 and E^1 to E_6^4

(iii) $H^1\ (X) \rightarrow H^5(X), [\alpha] \rightarrow [\alpha] \cup [\frac{1}{2}w^2]$ maps A^1 to A^5 and E^1 to E^5

proof: (i) is obvious, since * maps $A^m \leftrightarrow E^{6-m}$.

$[\alpha] \rightarrow [\alpha] \cup [\Omega]$ is a bijection $H^1\ (X) \rightarrow H_6^4(X)$. It maps A^1 into A^4 and E^1 In to E^4. It follows that $A^1 \rightarrow A_6^4$ and $E^1 \rightarrow E_6^4$ are both surjective and that $H_6^4(X)$ splits as $A_6^4 \oplus E_6^4$. $H_6^2\ (X)$ splits too by (i).

(iii) easily follows from (i) and (ii) in the same way.

Lemma 1.8. Let M be a Ricci-at EAC manifold:

(i) If M has a finite normal cover homeomorphic to a cylinder then M or a double cover of M is homeomorphic to a cylinder

(ii) If π_1(M) is infnite then M has a finite cover $\bar{M}$ with $b^1(\bar{M}) > 0$

Proof: (i) If $\bar{M}$ is a finite normal cover of M homeomorphic to a cylinder then it is isometric to a product cylinder Y × R. M is a quotient of Y ×R by a finite group A of isometries. The isometries are products of isometries of Y and of R (since they preserve the set of globally distance minimising geodesics{{y} × R: y ∈ Y}). The elements of A have finite order, so they must act on the R factor as either the identity or as reections. Therefore the subgroup B ⊆A which acts as the identity on R is either all of A, in which case M is the cylinder $\left(\frac{Y}{B}\right) \times$ R, or a normal subgroup of index 2, in which case $\left(\frac{Y}{B}\right) \times$ R is a cylindrical double cover of M.

(iii) Let $G_0 \subseteq \pi_1(M)$ be a nilpotent subgroup of finite index. G_0 is soluble, so the derived series G_i +1 = $[G_i, G_i]$ reaches 1. Therefore there is a largest i such that $G_i \subseteq \pi_1(M)$ has finite index. Let $\bar{M}$ be the cover of M corresponding to $G_i \subseteq \pi_1(M)$. $\frac{G_i}{G_{i+1}}$ is an infinite Abelian group, so has non-zero rank.

Theorem 1.9. Let M_- be M_+ with its orientation reversed and(φ_+ , φ_-) a matching pair of G_2-structures. If φ_+ and φ_- define the same metric then M_+ has a double cover isometric to a cylinder.

Proof. φ_- is a torsion-free G_2 -structure on M_+ which defines the same metric as φ_+.The matching condition for φ_+ and φ_- implies that the parallel section is asymptotic to $[\frac{\partial}{\partial t}]$. In other words either M_+ or a double cover of M_+ has a parallel vector field asymptotic to $\pm\frac{\partial}{\partial t}$ now this is impossible for a manifold with a single end, so M_+ has a double cover which is isometric to a cylinder. Result 1.10. Let $M_\pm$ denote the moduli space of torsion-free EAC G_2-structures on M± and N the moduli space of Calabi-Yau structures on their common cross-section X. We can define a subset $M_y \subseteq M_+ \times$M- consisting of pairs which have matching images in N.

While we can apply our understanding of M±and their relationship to N to show that M_y is a manifold, it is not an appropriate domain. The reason is that for a matching pair of points in the moduli spaces M_+, M_- there is some ambiguity in how to glue them.

Corollary 1.11. Let M be an asymptotically cylindrical manifold with non-negative Ricci curvature. Then the fundamental group π_1(M) has a nilpotent subgroup of finite index. M is homotopy equivalent to a compact manifold with boundary soπ_1(M) is finitely generated. Volume comparison arguments show that the volume of balls in the universal cover of M grows polynomially and this can be used to deduce that π_1(M) has polynomial growth

2 Main Result

if C = C7 + C14 is a skew-symmetric tensor, then the evolution of the skew-symmetric tensor P(C) under the ow equation:

$$\frac{\partial}{\partial t}\varphi_{ijk} = h_i^l\varphi_{ljk} + h_j^l\varphi_{ilk} + h_k^l\varphi_{ijl} + X^l\psi_{lijk}\ \ is\ given\ by:$$

$$\frac{\partial}{\partial t}(P(C))_{ij} = (P\left(\frac{\partial}{\partial t}C\right))_{ij} + 6\pi_7(\{h, C_{14}\})_{ij} - 6\pi_{14}(\{h, C_7\})_{ij} - 2\pi_7([X, C_{14}])_{ij} + 2\pi_{14}([X, C_7])_{ij}$$

Where π_7 and π_{14} denote the projections onto Ω_7^2 and Ω_{14}^2 respectively.
Proof. we see that $\frac{\partial}{\partial t}\left(C_{ab}g^{ap}g^{bq}\psi_{pgij}\right) equals$

$$\left(\frac{\partial}{\partial t}C_{ab}\right)g^{ap}g^{bq}\psi_{pgij}+2C_{ab}\left(\frac{\partial}{\partial t}g^{ap}\right)g^{bq}\psi_{pgij}+C_{ab}\,g^{ap}g^{bq}\left(\frac{\partial}{\partial t}\psi_{pgij}\right)$$
$$=(P\left(\frac{\partial}{\partial t}C\right))_{ij}-4C_{ab}h^{ap}g^{bq}\psi_{pgij}+C_{ab}h^{ap}g^{bq}(h_p^l\psi_{lqij}+h_q^l\psi_{plij})$$
$$+C_{ab}g^{ap}g^{bq}(h_i^l\psi_{pqlj}+h_j^l\psi_{pqil}-X_p\varphi_{qij}+X_q\varphi_{pij}-X_i\varphi_{pqj}+X_j\varphi_{pqi})$$
$$=(P\left(\frac{\partial}{\partial t}C\right))_{ij}-2C_{ab}h^{ap}g^{bq}\psi_{pgij}+h_i^l\big(P(C)\big)_{lj}+\big(P(C)\big)_{il}h_j^l$$
$$+2(C_{ab}X^bg^{bq})\psi_{pij}-6(C_7)_jX_i+6(C_7)_iX_j \qquad (1)$$

where we have used the skew-symmetry of C and of φ and relabeled indices to combine terms. The second term above can be written as

$$-2\,h_{al}g^{lm}C_{mb}g^{ap}g^{bq}\psi_{pqij}=-(h_{al}g^{lm}C_{mb}+C_{al}g^{lm}h_{mb})g^{ap}g^{bq}\psi_{pqij}$$
$$=-\{h,c\}_{ab}g^{ap}g^{bq}\psi_{pqij}=-P(\{h,c\})_{ij}=4(\pi_7\{h,c\})_{ij}-2(\pi_{14}\{h,c\})_{ij}$$
$$=4(\pi_7\{h,c_7\})_{ij}+2(\pi_7\{h,c_{14}\})_{ij}-2(\pi_{14}\{h,c_7\})_{ij}-2(\pi_{14}\{h,c_{14}\})_{ij}$$

Meanwhile the third and fourth terms of (1) become Combining these

$$\{h,P(C)\}_{ij}=\{h,-4c_7+2c_{14}\}_{ij}$$
$$=-4\,(\pi_7\{h,c_7\})_{ij}+2(\pi_7\{h,c_{14}\})_{ij}-4(\pi_{14}\{h,c_7\})_{ij}+2(\pi_{14}\{h,c_{14}\})_{ij}$$

expressions, after some cancellation we see that

$$\frac{\partial}{\partial t}\left(C_{ab}g^{ap}g^{bq}\psi_{pgij}\right)=(P\left(\frac{\partial}{\partial t}C\right))_{ij}+6\pi_7(\{h,C_{14}\})_{ij}-6\pi_{14}(\{h,C_7\})_{ij}$$

$$+2(C_{ab}X^bg^{bq})\psi_{pij}-6(C_7)_jX_i+6(C_7)_iX_j \qquad (2)$$

Consider now the third to last term above:

$$2\left(C_7(X_{ij})+C_7(X_{ij})\right)=2\,(-\frac{1}{2}\,[C_7\,,X]_{ij}-\frac{3}{2}\,(C_7)_iX_j\pm\frac{3}{2}\,(C_7)_iX_j\,[C_{14},X]_{ij}$$
$$=-[C_7,X]_{ij}+2\,[C_{14},X]_{ij}-3(C_7)_iX_j+3(C_7)_jX_i$$

Hence the final three terms of (2) are:

$$=-[C_7,X]_{ij}+2\,[C_{14},X]_{ij}+3(C_7)_iX_j-3(C_7)_jX_i$$
$$=-[C_7,X]_{ij}+2\,[C_{14},X]_{ij}-3(-\frac{1}{3}[C_7,X]_{ij}+\frac{2}{3}\,(C_7\times X)_{ij})$$
$$=-2[C_7,X]_{ij}+2\,[C_{14},X]_{ij}+2\,(C_7\times X)_{ij} \qquad (3)$$

now by using (1) and (2) and (3) the result is prove.

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